

(1) A Three-Layered Explanation of Paradoxes

(1.1) The Liar Paradox

'This sentence is false' (P). If this sentence is true, then it is false; if it is false, then it is true (Tarski, "The Concept of Truth," 1933). This paradox in fact contains three elements:

- 1) The negative judgment of P. It attempts to negate (change) the state of P and possesses a driving nature: motive force action.
- 2) P itself. It is an isolated, independent proposition: isolation action.
- 3) P itself being true. This means that for P, as a judgment, to judge other judgments, it must itself be true. Only on this basis can it judge other judgments (at least, we must default to this being the case). In other words, P itself being true is a prerequisite for making a judgment. This is the manifestation of P: manifestation action.

Let us see if these three constitute a no form integrated transformation:

- 1) If 'this sentence' (P itself) is false (the negative judgment of P), then 'this sentence is true' (P itself being true). That is, a judgment is made about P, and this judgment is true.
- 2) If 'this sentence' (P itself) is true (P itself being true), then the judgment can be made: 'this sentence is false' (the negative judgment of P).
- 3) To make a negative judgment about a proposition itself (the negative judgment of P), the proposition itself must be true (P itself being true); otherwise, it is not possible to make a judgment. From this, the proposition P itself can be determined.

This already contains the six transformations of a no form integrated transformation; the three do indeed constitute a no form integrated transformation. The original expression of the paradox is imprecise and confusing. Expressed in this way, it becomes clear.

In the subsection "Isolation Logic: The Unity of Formal Logic and Dialectical Logic," this paradox was explained using dialectical logic and was classified as Ind (Indeterminate). However, the explanation of it now using no form integrated transformation is clearer and more definite.

The explanation of the Liar Paradox using no form integrated transformation has already transcended the scope of formal logical reasoning; no form integrated transformation is a broader framework for reasoning. Therefore, it can be said that formal logic cannot explain such a paradox. Formal logic often regards paradoxes as anomalies or unsolvable problems. The framework of no form action theory regards them as a dynamic equilibrium.

- 1) That is to say, the explanation of this paradox is divided into three layers:
- 2) Formal logic considers this paradox to be an anomaly or unsolvable.
- 3) Dialectical logic can only describe the indeterminate state of this paradox.

No form integrated transformation reveals the essence of this paradox.

(1.2) Gödel's Incompleteness Theorems

The proof of Gödel's incompleteness theorems involves the construction of a self-referential proposition similar to the Liar Paradox: 'This proposition is unprovable' (Q) (Gödel, "On Formally Undecidable Propositions," 1931). In fact, its incompleteness embodies the incompleteness of formal logic, because within the scope of formal logic, no basis can be found to explain the self-transformation of this paradox. This clearly reveals the origin of Gödel's incompleteness theorems: within the scope of formal logic, formal logic cannot explain such a paradox. This also leads to the conclusion that to achieve completeness in reasoning, one must ascend to no form action theory.

Analyzing proposition Q (slightly different from P):

- 1) The negative judgment of Q: motive force action.
- 2) Q itself: isolation action.
- 3) Q itself being true: manifestation action.

Let us see if these three constitute a no form integrated transformation:

- 1) If 'this proposition' (Q itself) is unprovable (the negative judgment of Q), then what is actually being said is that Q itself is true.
- 2) If 'this proposition' (Q itself) is true (Q itself being true), then the judgment can be made: 'this proposition is unprovable' (the negative judgment of Q).
- 3) To make a negative judgment about a proposition itself (the negative judgment of Q), the proposition itself must be true (Q itself being true); otherwise, it is not possible to make a judgment. From this, the proposition Q itself can be determined.

(2) Multidimensional Dialectical Logic: The Extension of Isolation Dialectical Logic

I believe that isolation dialectical logic can be extended. For example, suppose there are three objects A_1 , A_2 , and A_3 . If A_1 is negated, one obtains non- A_1 : A_2 and A_3 . In this way, by negating A_1 , one obtains A_2 and A_3 (this is also determining A_2 and A_3 through A_1). By negating A_2 and A_3 , one obtains A_1 (this is also determining A_1 through A_2 and A_3). Through this opposition, they can be unified. Similarly, A_2 and ' A_1 and A_3 ', as opposites, can also be unified. A_3 and ' A_1 and A_2 ', as opposites, can also be unified. These three unifications are, in fact, the unity of A_1 , A_2 , and A_3 . In other words, this kind of unity possesses a multidimensional unity of opposites, or rather, multiple unities of opposites are synthesized together.

This can thus be extended to n opposing sides: $A_1, A_2 \dots A_n$. This is a multidimensional unity of opposites. We then say that this set of objects is in a multidimensional dialectical unity; they are unified into a single unified object. Of course, this can also be extended to an infinite number of opposing sides.

In this way, our understanding of 'unity' becomes more complex and multidimensional. Unity is no longer just the simple fusion of two opposites, but the complex orchestration of multiple oppositional dimensions, a dynamic equilibrium produced by different forces. This framework embodies the dynamism among the different dimensions of multidimensional dialectical logic. In traditional dialectics, the opposing sides are often seen as singular, but in this extension, the

relationship between the opposing sides is complex and compositional.

By introducing the concept of multiple opposing sides, this theory of multidimensional unity of opposites allows the problem of unity to be viewed from multiple perspectives and multiple dimensions. This not only enriches the application of dialectical logic but also provides a more flexible way of thinking for solving complex problems.

There is a special case: if A_1 and A_2 are in opposition, and A_1 and A_3 are in opposition, then we can unite these separate oppositions and treat them as a single opposition, thereby forming an opposition between A_1 and non- A_1 (A_2 and A_3).

(3) Applying Multidimensional Dialectical Logic to Set Theory Below

(3.1) The Definition of a Set

Since modern mathematics is founded on set theory, explaining the 10 axioms in the ZFC axiomatic set theory system is also to explain set theory, and thus to provide a foundation for mathematics, a philosophical foundation for modern mathematics.

First, we use multidimensional dialectical logic to define a set: for a group of objects A_i (where 'i' is not predetermined), if this group of objects is in a multidimensional dialectical unity, then this group of objects is unified into a single object A. We call this A a set. Since the set A is in a dialectical unity, its various elements are all independent and are mutually determined ('mutually determined' means that they define each other through their mutual oppositional relationship). Therefore, they also possess the property of independence: affirming oneself and negating the other. If there is only one object 'a', and it itself is determinate, then it can also form a set, because it itself determines itself. This situation is 'a is a', a self manifesting a self. Note, the elements of set A are mutually isolated (or mutually determined). This means that the elements of A can be either absolutely isolated or relatively isolated. Relative isolation means that the elements of a set are mutually isolated, but outside of the set, the elements may not possess isolation.

This definition of a set seems reasonable. However, there is still a problem. If we do not need to obtain a certain element from set A, then this definition of a set is sufficient. But if we need to be able to obtain a certain element from set A, then this definition of a set lacks this function. This definition of a set does not tell us how to obtain a certain element from set A; it only says that according to dialectical logic, the two opposing sides can be mutually transformed and determined. For example, in the subsection "Isolation Logic," we know that 'true and false can be unified into Ind (Indeterminate)', but this does not tell us how to obtain true or false from Ind. The acquisition of true or false is achieved through not(Ind). Therefore, it can be said that if we need to be able to obtain a certain element from set A, then the scope of multidimensional dialectical logic is greater than the definition of a set.

Because the three no form actions possess identity, no single action can exist alone. Therefore, an isolated thing must be able to be manifested through a certain motive force process; otherwise, it would violate the identity of no form action theory. Therefore, for a set, as an isolated, independent individual, its manifestation is its structure. In other words, a set must be constructible; it must be structurally determinate, in order to satisfy the identity of no form

action theory. In this way, a specific element can be obtained from this construction and be manifested (for example: x is a natural number, $2x$ as a structure represents the set of even numbers; the set composed of real numbers greater than 0 and less than 1), and thus manifest the set itself. For a set with a finite number of elements, all elements can be obtained through the method of enumeration (the most basic method).

This is, in fact, the Axiom of Constructibility: that all sets are constructible. However, the ZFC set theory system does not have the Axiom of Constructibility, so this system needs to have the Axiom of Constructibility added to it. When the ZFC system has the Axiom of Constructibility added, it becomes ZFC+C (the final C is for Constructibility). This then allows sets to satisfy the identity of no form action theory.

Summary of the definition of a set: First, it must satisfy multidimensional dialectical logic. Second, it must satisfy the Axiom of Constructibility; one must be able to obtain all the elements of the set through the constructibility of the set.

One more issue must be noted. 'a' and 'non-a' are in opposition, so their unity is also determinate and independent. Because its opposing sides are both independent and determinate, and the unity is obtained by negating the opposition between 'a' and 'non-a', the unity is also, in the sense of dialectical logic, determinate and independent. This unity is jointly and completely determined by its opposing sides. This is the intrinsic independence of the unity (as has already been discussed in the subsection "Dialectical Logic").

(3.2) Deducing the ZFC Axioms

Let us first look at the intuitive explanation of the Axiom of the Empty Set: there exists an empty set $\varnothing = \{\}$. This set has no elements.

In the subsection "exploring philosophy with mathematics," it was discussed that no form becomes 'nothingness' in the world of isolation. From the perspective of the world of isolation, this 'nothingness' is something completely devoid of anything. This 'nothingness' expresses the fact that a concept has no attributes. When applied to the concept of a set, it is a set that has no elements. Such a set is then the empty set $\varnothing$. This, in fact, explains the Axiom of the Empty Set.

The Axiom of Constructibility used below requires that elements can be deterministically obtained from the structure of a set. Of course, there is also probabilistic acquisition of elements (which will be explored in the later subsection "probabilistic linear set"). For example, with $f = 2x$ as the structure for the set of even numbers, each time we obtain an even number, it is deterministic.

There is a kind of absolute isolation. As long as there is a difference between two objects, they possess isolation. They have an oppositional nature based on this difference (for example, if a and b are different, one can say 'a is not b'). This is, in fact, the isolation of the world of isolation. The elements of the sets discussed below are stipulated to possess this isolation. Next, I will use this definition of a set to deduce the 9 axioms of the ZFC axiomatic set theory system.

Note, the deduction of the axioms here is the most intuitive deduction. Axioms are, in their essence, intuitive. Due to the needs of mathematical operations, the axioms of set theory have become very abstract, but in essence, Immediacy is the most fundamental. If the Immediacy of

these axioms does not hold, then the axioms themselves do not hold.

(3.2.1) The Axiom of Choice

Intuitive explanation: One can select an element from a set.

In fact, I believe that no form action theory can be used to explain the intuitive Axiom of Choice. This requires a modification of the current concept of a set, by adding a special element 'negation' to the set as a motive force. Of course, this element is not a set member in the traditional sense; it does not directly participate in the operations of the set, but rather exists as an implicit 'motive force'. For example, $A = \{x_1, x_2, x_3, \text{not}\}$. In fact, the meaning of the concept of a set should be: the elements in a set are all isolated from each other; each element is independent of the others (as will be seen later, this independence does not mean they cannot have any relationship at all).

When we choose an element x_1 from set A , it is in fact to isolate x_1 . But this requires motive force action and manifestation action. This motive force action is 'not', and this manifestation action is the non- x_1 (x_2 and x_3) in set A . Non- x_1 is the reverse manifestation of x_1 . Choosing an element x_1 from set A already implies the negation of non- x_1 , because you did not choose non- x_1 . Choice is not just about picking something; it is also about rejecting other possibilities. This is a no form united transformation (in fact, x_1 , non- x_1 , and 'not' also constitute a no form integrated transformation). This is also isolation dialectical logic.

It is now clear: the reason x_1 can be chosen from set A is because x_1 can be manifested through the motive force 'not' and non- x_1 . This conforms to no form united transformation. In other words, the reason x_1 can be chosen is because a no form united transformation has been implemented.

Note, the Axiom of Choice only says that an element can be selected from a set. This can only express the dialectical relationship among the elements in a set; it does not say how to obtain an element from a set. Therefore, it cannot replace the Axiom of Constructibility; they are different.

Reflection on the Axiom of Choice:

Mathematical formal expression: For a family of non-empty sets $\{A_i \mid i \in I\}$ (where I is an arbitrary index set), which form a set $A = \{A_i \mid i \in I\}$, there exists a choice function $F: I \rightarrow \bigcup_{i \in I} A_i$, such that for each $i \in I$, $F(i) = a_i \in A_i$. These elements $\{a_i \mid i \in I\}$ form a set 'a'.

In other words, the Axiom of Choice has two functions. The first is to select a certain element from a set. The second is that these selected elements can form a set.

The first function is my intuitive explanation of the Axiom of Choice using multidimensional dialectical logic.

The second function can also be obtained from my definition of a set:

Due to the first function, we can arbitrarily choose a set, say A_x , and arbitrarily choose an element a_x in A_x . Then, we can arbitrarily choose another set, say A_y , and arbitrarily choose an element a_y in A_y (assuming a_x and a_y are two different elements). Since A_x and A_y are both sets, a_x and a_y are both independent. Since we have assumed they are different, they are therefore in

opposition. Due to the arbitrariness of a_x and a_y , ' a ' = $\{a_i \mid i \in I\}$ satisfies the multidimensional dialectical logic in the definition of a set.

Since A is a set, A satisfies the Axiom of Constructibility, and ' a ' inherits the structure of A . This inheritance can be understood in this way: the set A is constructible, so its element A_x can be obtained. Then, an element a_x can be arbitrarily obtained from A_x (the set A_x is also constructible). In this way, a concrete set ' a ' can be obtained. For this concrete ' a ', we can obtain all the elements of set ' a ' by obtaining each element of A , and then obtaining each element of each element of A . Therefore, ' a ' satisfies the Axiom of Constructibility. Therefore, ' a ' completely satisfies the definition of a set, so it is a set. If a_x and a_y were the same, they could be merged into a single element, which would not affect $\{a_i \mid i \in I\}$ being a set. In this way, the second function is deduced. Of course, in the second function, it is not necessary to specify which a_x it is, but rather any one in A_x . The second function only expresses the existence of the set ' a '.

We see that this axiom can be separated. Therefore, we can retain the first function in the Axiom of Choice and take the second function as an inference from the first. This can well maintain the clarity and conciseness of the axiom.

Natural induction (mathematical induction) is one of the most basic proof tools in mathematics, widely used in fields such as number theory and algebra. Traditionally, it has been regarded as a self-evident axiom or an intuitive mathematical principle, lacking a clear philosophical origin. Since natural induction can be deduced from the well-ordering theorem, and the well-ordering theorem is equivalent to the Axiom of Choice, natural induction can therefore be deduced from the Axiom of Choice. Since the Axiom of Choice can be deduced through no form action theory and multidimensional dialectical logic, no form action theory and multidimensional dialectical logic can therefore deduce natural induction. This provides a philosophical basis for natural induction, elevating it from an 'intuitive assumption' to a 'result of philosophical deduction'.

(3.2.2) The Axiom of Extensionality

Intuitive explanation: A set is completely determined by its elements. If two sets contain the same elements, then they are equal, and vice versa.

Since we have already explained the Axiom of Choice using no form action theory, we can now use no form action theory to explain the 'Axiom of Extensionality'.

Let us assume that two sets, A and B , contain the same elements. According to the Axiom of Choice, we can select any element ' x ' from a set. By using negation, we can obtain the other elements in the set ($\text{non-}x$). Then, ' x ' is not ' $\text{non-}x$ '. Then, according to the assumption, since sets A and B contain the same elements, we can arbitrarily select the same element ' x ' from A and from B . Then, the elements in $\text{non-}x$ in A and the elements in $\text{non-}x$ in B should be the same. Otherwise, there would be an element xx in the $\text{non-}x$ of A but not in the $\text{non-}x$ of B , or vice versa. Since xx is not x , xx is in A but not in B , which contradicts the assumption.

In this way, for any ' x ' in A , the same ' x ' can also be obtained in B , and at the same time, the same $\text{non-}x$ can also be obtained. Due to the arbitrariness of ' x ', according to the definition of a set, the multidimensional dialectical logic in the definition of A and B is the same. Obtaining the element ' x ' from A also means one can obtain ' x ' from B , and vice versa. This shows that the acquisition of

set elements is independent of the specific construction of the set. That is, it is sufficient that a set has a construction; it is sufficient that one can obtain elements from the set. Therefore, even if two sets have different constructions, as long as they have the same elements, then the two sets are the same. Therefore, $A = B$.

Conversely, if $A = B$, then according to the definition of a set, they have the same opposing sides, which means they have the same elements.

This proves that a set is completely determined by its elements. In other words, if two sets are not equal, then they must have different elements.

(3.2.3) The Axiom of Pairing

Intuitive explanation: If there are two independent objects, they can form a set.

According to my definition of a set, the Axiom of Pairing is very simple. For two different objects 'a' and 'b' that are independent, they can mutually negate and mutually determine each other. Therefore, $A = \{a, b\}$ satisfies the multidimensional dialectical logic in the definition of a set. Moreover, A has only two elements, its structure is determinate, and all its elements can be obtained through enumeration. Therefore, A satisfies the Axiom of Constructibility. Therefore, according to the definition of a set, A is a set. If the two objects are the same (e.g., object 'a' and object 'a'), since 'a' itself is an independent object, it can become a set with one element, $\{a\}$.

(3.2.4) The Axiom of Regularity

Intuitive explanation: 1) A set cannot contain itself. 2) The elements of a set cannot form an infinitely descending chain (an element A_0 in set A contains A_1 , which contains A_2 , which contains A_3 , and so on, forever).

The first part of this axiom has already been explained in the subsection "Formal Logic". Now, it can still be explained using the definition of a set.

1) A set cannot contain itself: Suppose there is a set A and another object b, and A and b are mutually independent. Then, if we negate A, b is non-A. Then they can be unified into a set B. The question now is whether B can equal A. If $B = A$, then by negating A, b cannot be non-A, because b is still in A. Therefore, A and b cannot form an opposition, and thus cannot be unified into $B = A$. In other words, a set A cannot contain itself. The b obtained by negating A can only exist in a set B that is not equal to A, in order for A and b to be unified into B.

Note, b can be a part of A. For example, $B = \{b, \{b\}\}$, $A = \{b\}$. B is a valid set. If we negate $\{b\}$, we can obtain b (non- $\{b\}$). This b that is obtained is the b in B, not the b in $\{b\}$. Although they are both 'b', their positions are different, and thus they become different. They can be seen as copies of 'b' in different places. Therefore, B is a valid set. This shows that the 'b' generated by 'negation' must be able to go beyond the self-containment of A.

2) Infinitely descending chains: If the final endpoint of this infinitely descending chain is the set B, then because it is an infinitely descending chain, the situation $B \in B \in B \dots$ would occur, continuing forever. Therefore, B would also be an element of B, which would then violate the principle that a set cannot contain itself.

For the elements $A_0 \in A_1 \in A_2 \in A_3 \in \dots$ in a set A to continue forever, without end, in this situation, A_0 as an element of A is in fact infinitely nested (similar to $A = \{A_0\}$, $A_0 = \{A_1\}$, $A_1 = \{A_2\} \dots$). Since sets have no measure (for example, a line segment has length; length is the measure of the line segment), all sets are equal. Therefore, this nesting can never have a limit. Then, any element in this nesting will depend on the independent existence of the element inside it for its own existence. Since no succeeding element can be determined to exist independently, then no element can exist independently, and no element is determinate. Therefore, A_0 is not determinate, so such a set is meaningless. Because dialectical logic requires that both opposing sides be able to affirm themselves, and A_0 cannot affirm itself, A does not conform to the definition of a set.

For a self-containing set like $A = \{A\}$, it can also be seen as the nesting situation described above. According to the above explanation, A does not conform to the definition of a set.

The Axiom of Regularity is often described as a somewhat ad hoc technical axiom to avoid paradoxes, rather than a principle with a deep philosophical foundation. However, using dialectical logic, an elegant explanation can be obtained.

(3.2.5) The Axiom of Union

Intuitive explanation: For a family of sets, where this family also forms a set A , combining their elements together can form a set.

For simplicity, let us consider two sets, A_x and A_y . Since they are both sets, their elements are all independent. Therefore, any two different elements among them are in opposition.

1) Assume that any x belongs to A_x but not to A_y . Then, the other elements in A_x are not x , and all the elements in A_y are not x . That is to say, the non- x in A_x and A_y is determinate.

2) Assume that any x belongs to A_y but not to A_x . The logic is the same as 'assume x belongs to A_x but not to A_y '.

3) Assume that any x belongs to both A_x and A_y . Then, the other elements in A_x are not x , and the other elements in A_y are not x . That is to say, the non- x in A_x and A_y is determinate.

In other words, all the elements in sets A_x and A_y satisfy the multidimensional dialectical logic in the definition of a set. Moreover, since A is a set, each of its elements can be obtained. Thus, each element of each element of A can be obtained, which means all the elements of A_x and A_y can be obtained. Therefore, the union of A_x and A_y satisfies the Axiom of Constructibility. Therefore, according to the definition of a set, their union is a set.

(3.2.6) The Axiom of Separation (or Specification)

Intuitive explanation: The Axiom of Separation is that any subset of a set is a set.

For any set A , let AA be any subset of A . Then for any element ' x ' in AA , ' x ' is also an element in A . According to my definition of a set, then, in A , the elements that are not ' x ' are all 'non- x '. Therefore, any elements in AA that are not ' x ' are also 'non- x '. Consequently, both ' x ' and 'non- x ' in AA are independent and determinate. As opposites, they can be unified into the set AA . In other words, AA satisfies the multidimensional dialectical logic in the definition of a set.

Moreover, since A is a set, all its elements can be obtained, and therefore all the elements of AA can also be obtained. Thus, AA satisfies the Axiom of Constructibility. Therefore, according to the definition of a set, AA is a set.

This shows that in a multidimensional dialectical logic, any number of objects within it can also be in a unity of opposites, and can thereby constitute a valid unity.

(3.2.7) The Axiom of Replacement

Intuitive explanation: For any set A and a function $f(x)$, on the premise that $f(x)$ is defined for every x in A , the range of f , B , is also a set.

Let us first assume that the function f is a one-to-one correspondence from A to B . Since f is a function, all the elements of B are independent and determinate.

If there is only one element, bb , in B , then B is a set, because bb can only be determined by itself ('a self is itself'), and its self-determination determines this set.

If there are more than one element in B , we can arbitrarily choose an element ' b ' in B . According to the function f , there must be a unique element ' a ' in A that corresponds to it. Then for any element a_1 in A that is not ' a ' (that is, non- a), since f is a one-to-one correspondence, it must be that $f(a_1)$ is not equal to ' b '. Therefore, the elements in B other than ' b ' are not ' b '. Consequently, both ' b ' and 'non- b ' are determinate. Their opposition can be unified into B . Therefore, B satisfies the multidimensional dialectical logic in the definition of a set. Moreover, the function f is a one-to-one correspondence from A to B . Therefore, A and B have the same structure. That is, by obtaining all the elements of A , one can, through f , obtain all the elements of B . Therefore, B satisfies the Axiom of Constructibility. Therefore, according to the definition of a set, B is a set.

For the case where the function f is not a one-to-one correspondence from A to B , according to the Axiom of Separation, any subset of A is a set. Therefore, for variables with the same function value, we can take only one of them and form a subset of A , AA , with the single-valued variables (applying the Axiom of Choice). Thus, the domain of the function f can be changed to AA , and in this way, f becomes a one-to-one correspondence from AA to B . Therefore, applying the previous explanation, B is a set.

(3.2.8) The Axiom of Power Set

Intuitive explanation: The set of all subsets of a set forms a set.

For any set A , we can construct its power set $P(A)$, which is the set composed of all possible subsets of A . Simply put, the Axiom of Power Set states: no matter what kind of set you have, you can always generate a new set that contains all the subsets of the original set.

According to the Axiom of Separation, any subset of a set A is also a set. Since all sets are unities, the subsets of A are all independent and determinate. Moreover, the elements of $P(A)$ are composed of different subsets of A (according to the Axiom of Extensionality, these subsets are different because they have different elements). Therefore, any two different elements in $P(A)$ are in opposition, and the elements of $P(A)$ can be unified into $P(A)$ through opposition. Therefore, $P(A)$ satisfies the multidimensional dialectical logic in the definition of a set. Since any subset A_x of A is a set, this in itself is a determinate structure. In other words, all the elements of

$P(A)$ can be obtained by generating subsets from A . Therefore, $P(A)$ satisfies the Axiom of Constructibility. Therefore, according to the definition of a set, $P(A)$ is a set.

This shows that in a multidimensional dialectical logic, as long as its objects possess absolute isolation, the combination of multiple objects and another combination of multiple objects can also be in a unity of opposites.

(3.2.9) The Axiom of Infinity

Intuitive explanation: In ZFC set theory, there exists an infinite set, whose construction begins with the empty set $\emptyset$ and is recursively generated through the successor operation $x \cup \{x\}$. For example, $A = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \dots\}$. This type of set has already been explained in the context of the Axiom of Regularity; it satisfies the multidimensional dialectical logic in the definition of a set (explained in the context of the Axiom of Regularity using $B = \{b, \{b\}\}$). Moreover, through this recursive structure, all the elements of A can be obtained. Therefore, A satisfies the Axiom of Constructibility. Therefore, according to the definition of a set, A is a set.

This shows that within the framework of no form action theory, infinity is not a concept that needs to be mysteriously postulated, but a legitimate logical structure that can be continuously generated through dialectical unity (such as the successor operation) and the principle of constructibility.

Adding the Axiom of Constructibility to the ZFC system is compatible with the system itself; no contradiction is generated. Moreover, it makes the explanation of these axioms clearer, more reasonable, more natural, and more rigorous. For example, in the Axiom of Choice, for the family of sets $A = \{A_i \mid i \in I\}$ (where I is an arbitrary index set), since A is a set, all its elements can be obtained according to its construction. And thus, according to the first function of the Axiom of Choice, an element can be obtained from each element of A to form a set. If we could not obtain all the elements of A , we would not know how to choose elements from an infinite number of sets. Without the Axiom of Constructibility, this can only be entirely relegated to an axiom. This, in fact, tells us that the Axiom of Constructibility allows us to extend from finite means to infinite means. This extension allows my intuitive deduction of these 9 axioms to be naturally extended to a rigorous mathematical expression.

My method provides an intuitive explanation for the 9 axioms of ZFC and unifies them under the framework of 'dialectical logic', reshaping these seemingly fragmented axioms into an organic system that logically and necessarily grows out of the philosophical principle of 'multidimensional dialectical unity'. My explanation shows that these 9 axioms can be seen as a unified dialectical system, mutually consistent, with no contradiction found among them. This shows that these axioms, as a whole, conform to the fundamental principles of opposition, negation, and unity of dialectical logic.

Gödel's second incompleteness theorem shows that the consistency of ZFC cannot be proven internally; a stronger external axiom is required (Gödel, "On Formally Undecidable Propositions," 1931). However, my method clearly steps outside of traditional formal logic, because it introduces meta-logical concepts that formal logic itself cannot handle, such as dialectical unity and no form action, and examines the foundations of these axioms from within a broader ontological framework.

These 9 axioms are the methods of operation on sets. Their deduction has all used multidimensional dialectical logic, which shows that these axioms are different manifestations of multidimensional dialectical logic. Each axiom embodies the process of opposition and unity. In fact, these 9 axioms are the symbolic (mathematical) expression of multidimensional dialectical logic; this is the essence of these axioms. They embody the process of dialectical logic in a symbolic way. Therefore, these 9 axioms are ultimately expressed as multidimensional dialectical logic itself.

This redefinition of the set elevates the set from a static mathematical object to a dialectical philosophical entity, revealing the generative mechanism behind set theory. This not only explains the mathematical reasonableness of the ZFC axioms, but also provides them with an ontological and epistemological philosophical foundation, achieving the goal of 'exploring philosophy with mathematics'. ZFC set theory is not only the foundation of mathematics, but also the mathematized expression of dialectical logic. This is consistent with the viewpoint I mentioned in the subsection "exploring philosophy with mathematics": 'the process of no form transforming into form must necessarily be accompanied by a mathematical structure'. The essence of mathematics is redefined as a dynamic generative process. In this way, philosophy and mathematics are connected.

Now that we have used no form action theory and dialectical logic to explain all the axioms in ZFC+C set theory, we have successfully provided a foundation for set theory. No form action theory and dialectical logic have become the foundation for set theory, and thus the foundation for mathematics. Mathematics is no longer a formal system detached from philosophy, but a direct embodiment of philosophical logic. Furthermore, since philosophy has become the foundation for all the axioms in ZFC+C set theory, these axioms can no longer be called axioms. They are no longer intuitive, no longer self-evident, but can be deduced using no form action theory. Mathematics is no longer a static axiomatic system, but the result of transformation through a dialectical process. Mathematics becomes the symbolic expression of philosophy, and philosophy becomes the internal logic of mathematics.

Since mathematical operations require the use of formal logic, mathematics is a combined application of dialectical logic and formal logic. This combination is a typical application of the two logics. Gödel's incompleteness theorems are in fact showing that the use of formal logic alone is incomplete. Dialectical logic and formal logic must be integrated into one, and both must be used simultaneously to completely construct the mathematical system. Now that the axioms of ZFC+C have been uniformly explained using no form action theory and dialectical logic, mathematics will enter an era where the two logics are integrated into one. This is a major revolution in the history of mathematics. Dialectical logic is also a formalized logic, so the method of proving and deducing the consistency of a system based only on formal logic as the standard has its limitations. As has been argued in the subsection "Dialectical Logic," dialectical logic and formal logic are complementary logics; using only one of them is incomplete.

For a long time, metaphysics has often been considered too abstract, empty, and disconnected from specific disciplines and the real world. Since metaphysics is the study of the laws 'behind' specific laws, axioms (especially the axioms of mathematics) should be the objects of study for metaphysics. Because these axioms cannot be proven within the systems of their specific

disciplines, they therefore belong to the domain of metaphysical research. This provides a direction for metaphysical research: to study the axioms of a specific field (such as physics, psychology, sociology, etc.), and thereby to be able to explain the origin and essence of these axioms from a philosophical perspective, and in turn to provide a foundation for these fields with philosophy. In this way, philosophy and specific fields are connected, making philosophy a universal tool for 'founding disciplines'. This ensures that metaphysics is no longer a castle in the air, but can directly serve the foundational research of specific disciplines, bringing about an unfold-manifestation of its powerful applied value.

(4) Proving the Continuum Hypothesis Using No Form Action Theory and Dialectical Logic:

Gödel's first incompleteness theorem states that within a consistent formal system that contains elementary arithmetic, there exists a proposition P that can neither be proven true nor false (Gödel, "On Formally Undecidable Propositions," 1931). Strictly speaking, such a proposition appears within the scope of formal logic. According to the isolation logic I have constructed, the proposition P can be set to: Ind (Indeterminate). In this way, Gödel's incompleteness theorems become theorems of completeness. In fact, this is precisely the state we need, because the state of Ind is an indeterminate dialectical state. Therefore, in this situation, if a certain proposition appears in the Ind dialectical state, it means that it needs to be explained or handled using a theory that transcends formal logic.

ZFC set theory is such a formal system. Within it, there must be a proposition like P . For such a proposition, we must certainly use a theory that transcends formal logic to explain it. In the Trolley Problem in the subsection "Isolation Logic," a result of the Ind state was obtained, and in the end, the problem was solved through a dialectical method of negating one choice to obtain another.

Description of the Continuum Hypothesis: In ZFC set theory, there is no cardinal number between the countably infinite cardinal number (the cardinality of the set of natural numbers), $\aleph_0$, and the cardinal number of the continuum (the cardinality of the set of real numbers), $2^{\aleph_0}$. This is often denoted as CH.

(4.1) Gödel proved that CH is consistent relative to the ZFC axiom system. This means that within the ZFC axiom system, CH cannot be proven false. In other words, CH is consistent with the ZFC axiom system and does not lead to a contradiction (Gödel, The Consistency of the Continuum Hypothesis, 1940).

Gödel constructed a special set-theoretic universe (L) , called the 'constructible universe'. In (L) , all sets can be generated from the 'empty set' through a recursive construction method.

Because (L) is a model of ZFC (it satisfies all the ZFC axioms), and in (L) , CH is true, therefore CH does not contradict ZFC. Therefore, CH cannot be proven false within ZFC; otherwise, the model (L) would be inconsistent.

Steps in Gödel's Construction of the Constructible Universe (L)

(This is not a detailed mathematical proof, but a demonstration of the steps.)

Step 1: Constructing the set of natural numbers

Starting from $L_0 = \emptyset$:

$L_1 = \{\emptyset\}$ (representing 0)

$L_2 = \{\emptyset, \{\emptyset\}\}$ (representing 0, 1)

$L_3 = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$ (representing 0, 1, 2), ...

Until we reach $L_\omega = \bigcup_{n < \omega} L_n = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \dots\}$:

This contains all natural numbers (as von Neumann ordinals) and has cardinality $\aleph_0$ (countable infinity).

Step 2: Beginning the construction of constructible subsets of $P(\omega)$

$L_{\omega+1} = \text{Def}(L_\omega)$:

Contains all definable subsets of L_ω , such as "the set of even numbers" $\{0, 2, 4, \dots\}$.

Does not contain non-constructible subsets of $P(\omega)$ (e.g., random reals).

The cardinality is still $\aleph_0$ (countable), but this begins to build the foundation for $P(\omega)$.

Step 3: Completing the construction of $P(\omega)$

$L_{\omega_1} = \bigcup_{\alpha < \omega_1} L_\alpha$:

ω_1 is the first uncountable ordinal. L_{ω_1} contains all subsets of $P(\omega)$ that are constructible in (L) .

In the model $V = L$, we have $P(\omega) = \{X \subseteq \omega \mid X \text{ is definable in } L\}$.

Cardinality calculation:

Each $X \in P(\omega)$ is defined by an ordinal $\alpha < \omega_1$ (where $|\omega_1| = \aleph_1$) and a countable formula.

Thus, $|L_{\omega_1}| = |\omega_1| \cdot \aleph_0 = \aleph_1$.

The axiom $V = L$ excludes non-constructible subsets (such as those in models where $2^{\aleph_0} > \aleph_1$).

Since $2^{\aleph_0} \geq \aleph_1$ (by Cantor's theorem), it follows that $2^{\aleph_0} = \aleph_1$ (the Continuum Hypothesis, CH).

(4.2) Paul Cohen's Proof via Forcing that CH is Not Provable in ZFC

(Cohen, Independence of the Continuum Hypothesis, 1963)

Step 1: Select an initial model

Take a countable transitive model (M) that satisfies ZFC and contains ω (with $|\omega| = \aleph_0$) and ω_1^M (with $|\omega_1^M| = \aleph_1^M$).

Step 2: Define the forcing conditions

Let $P = \{p \mid p: \omega \times \omega_2^M \rightarrow \{0, 1\}, \text{dom}(p) \text{ is finite}\}$.

A condition (p) is a finite partial function, where ω_2^M is the cardinal $\aleph_2^M$ in (M) .

The order is defined by $p \leq q$ if $q \subseteq p$ (p is an extension of q).

Step 3: Introduce a generic set (G)

$G \subseteq P$ is a generic filter, meaning it intersects every dense subset (D) of P that exists in (M) .

Define a set of new real numbers $f_{G,\alpha}: \omega \rightarrow \{0,1\}$ for each $\alpha < \omega_2^M$:

$$f_{G,\alpha}(n) = 1 \quad \text{if} \quad \exists p \in G, p(n, \alpha) = 1.$$

Step 4: Extend the model to $(M[G])$

$(M[G])$ is the extension of (M) by (G) . It is also a model of ZFC.

Cardinals in the new model:

Cardinals are preserved: $|\omega|^{M[G]} = \aleph_0$, and $|\omega_1^M|^{M[G]} = \aleph_1^{M[G]}$.

The powerset of ω is now larger: $|P(\omega)|^{M[G]} \geq |\omega_2^M| = \aleph_2^{M[G]} > \aleph_1^{M[G]}$.

Therefore, in $M[G]$, $2^{\aleph_0} > \aleph_1$, and CH is false.

Step 5: Conclusion on ZFC Consistency

Since $(M[G])$ satisfies ZFC, and ZFC is consistent, then CH is unprovable in ZFC.

Thus, CH can neither be proven true nor false; this is the dialectical state Ind.

From the perspective of formal logic, Cohen's proof is not problematic. However, from the perspective of no form action theory, Cohen's proof is problematic. In his proof, Cohen assumes the existence of (M) and (G) without constructing the specific elements of $(M[G])$, which does not satisfy my definition of a set. A set must be constructible (that is, a set must satisfy the Axiom of Constructibility), just as Gödel did in his proof. Therefore, Cohen's proof is problematic.

The essence of a proof is to satisfy identity, whether it is the identity of formal logic, the identity of dialectical logic, or the identity of no form action theory. In fact, the identities of these two logical systems both belong to the identity of no form action theory. Therefore, in essence, a proof is to satisfy the identity of no form action theory. This in fact expands the scope of mathematical proof. It is no longer limited to the identity of formal logic. A proof not only requires satisfying the laws of formal logic, but also dialectical unity, and also constructible manifestation.

This precisely embodies the three no form actions that a proof possesses: the laws of formal logic correspond to the static isolation action (formal logic emphasizes distinction, definition, and static structure); dialectical unity corresponds to the dynamic motive force action; and constructible manifestation corresponds to the immediate manifestation action. In other words, a proof must simultaneously satisfy these three identities; none can be missing.

In traditional proofs of formal logic, people, through experience (for example, by constructing axioms), have ensured that the identity of dialectical logic and the identity of manifestation are satisfied in most cases. However, when it comes to the most fundamental field of mathematics,

set theory, formal logic has clearly exposed its shortcomings.

If the ZFC system has the Axiom of Constructibility added to it, then Gödel's proof has already proven CH. Because, according to Gödel's proof, the set he constructed satisfies the Axiom of Constructibility, and thus satisfies the identity of manifestation. And he exhausted all constructible sets. His proof shows that CH and ZFC have no contradiction, which satisfies the identity of formal logic. Moreover, his construction satisfies ZFC. Since the axioms of ZFC satisfy multidimensional dialectical logic, his construction satisfies the identity of dialectical logic. More precisely, since the axioms in ZFC specify all the operations on sets, the set he constructed satisfies the identity of dialectical logic in its operations. Therefore, Gödel's proof satisfies all the identities. Consequently, ZFC+C plus Gödel's proof has already proven CH. Therefore, there is no set with a cardinal number between that of the natural numbers and the real numbers.

From the perspective of no form action theory, ZFC only becomes a complete axiomatic system when it becomes ZFC+C (and with the addition of formal logic, it already completely satisfies the identity of no form action theory). The reason why CH could not be proven is because ZFC lacks the Axiom of Constructibility.

For mathematical proofs, within a consistent formal system containing elementary arithmetic, Gödel's incompleteness theorems have already proven that there are propositions that can be judged neither true nor false within the scope of formal logic. For such a proposition, if it is not contradictory within this system, that is, it satisfies the identity of formal logic, then it is necessary to see if it satisfies the identity of dialectical logic. If it does, then it is further necessary to see if it satisfies the identity of manifestation. If all are satisfied, then the proposition is correct.

(5) A Complete Framework for Reasoning

We use reasoning to obtain determinate and reliable conclusions. But why is reasoning itself reliable? What is the essence of reasoning? Usually, people believe that reasoning is reliable based on experience and practice. They just think that 'doing it this way is the best', but they have failed to provide a convincing theoretical explanation for it and lack an understanding of the essence of reasoning. A proof is a special kind of reasoning. My explanation of 'proof' can be entirely used as an explanation for 'reasoning'. The essence of reasoning is also to satisfy identity. Our usual reasoning is based on formal logic. Like proof, we can also extend reasoning. Reasoning must satisfy three identities: the identity of formal logic, the identity of dialectical logic, and the identity of manifestation.

The precise explanation that the essence of reasoning is to satisfy identity:

- 1) Reasoning must satisfy the three fundamental laws of formal logic, and these three fundamental laws constitute a no form integrated transformation. A no form integrated transformation must satisfy the identity of no form. Therefore, reasoning must satisfy the identity of formal logic.
- 2) Similarly, dialectical logic also has three fundamental laws: the law of negation, the law of dichotomy, and the law of unity. They also constitute a no form integrated transformation.
- 3) It must satisfy the identity of manifestation. To construct a concept or a symbolic object, one

must be able to obtain it and manifest it. For set theory, this means that a set must have a manifested structure, to embody that the elements of a set can be manifested through the motive force process of selection.

4) On a more fundamental level, it must satisfy the logic of identity, which includes: no form united transformation, no form integrated transformation, and the no form trinity. This is a more fundamental logic; it transcends formal logic and dialectical logic.

This is a framework for reasoning that satisfies the absolute identity of no form. Because the absolute identity of no form is the most fundamental ground, and every thing must follow this ground, this is therefore a complete framework for reasoning, and reasoning conducted using this framework is reliable.

Not only that, but this framework for reasoning is also universally applicable, because it contains multiple types of reasoning and is suitable for various fields, for instance, philosophy, the humanities and social sciences, mathematics, physics, logic, art, and so on. Its scope of reasoning is much larger than that of formal logical reasoning.

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